FLUID MECHANICS

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Keywords: continuum hypothesis, continuity equation, Navier-Stokes equations, Stokes' equation, Euler's eqynolds number, laminar flow, turbulent flow, Hagen-Poiseuille flow, Newton's law of viscosity, viscous stress, shear stress, normal stress, control volume, form drag, frictional drag, drag force, drag coefficient, terminal velocity, Stokes' settling velocity, Richardson-Zaki correlation

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Summary

Most chemical processing operations involve the motion of gases and/or liquids. A knowledge of fluid dynamics is therefore essential in the field of chemical engineering. This contribution presents an introduction to fluid dynamics. We begin with a derivation of the basic physical laws governing the behavior of fluids in motion. These include the equations describing conservation of mass and Newton's second law of motion. In their general form, these equations are complex, non-linear partial differential equations, which need appropriate boundary conditions. We discuss the most common types of boundary and initial conditions for fluid flow. We highlight how a variety and complexity of flow patterns arise in consequence of the governing laws. Using dimensional analysis, we investigate the role of the Reynolds number on the type and structure of the flow. The governing equations are applied to the problem of laminar flow in a circular pipe; the Hagen-Poiseuille velocity profile is derived. Finally, we analyze the motion of a fluid around a small, solid sphere. The drag force the fluid exerts on the sphere is calculated for laminar flow and stated as Stokes' law. This result is used to derive an expression for the velocity of an isolated solid sphere settling under gravity in a fluid. The terminal velocity of a particle settling in a suspension is also analyzed.

1. Introduction

Understanding the motion of fluids is important in a vast array of contexts. These range from natural phenomena such as river flow, ocean currents, and air motions which affect the weather, to the human uses of fluids, including our water supply and sewerage systems, the forces on airplanes, the design of turbines and pumps, and of course breathing and blood flow in our bodies.

Both gases and liquids are classified as fluids. Fluids differ from solids in that they deform continuously under the action of a tangential or shear force, however small. Thus, when shear forces are present in a fluid, it moves and a variety of flow patterns may ensue. Certain physical properties of a fluid are intimately related to its behavior in motion. These include, among others, the density and the viscosity of the fluid.

The study of fluids in motion constitutes the field of *fluid dynamics*. Fluid dynamics is an interdisciplinary field, which involves the work of applied mathematicians, physicists and engineers. A knowledge of fluid dynamics is also essential in the field of chemical engineering. Indeed, most chemical processing operations involve one or more fluid phases in motion. Examples of such operations include mixing, separation and reaction processes in the chemical, pharmaceutical, food and petroleum industries. Fluids abound in industry for two main reasons. First, at typical operating conditions, many materials are gases or liquids. Second, fluids can be easily transported, mixed and separated, so it is usually more cost-effective to work with fluids rather than with solids. Furthermore, some operations involving solids are designed to mimic the behavior of a fluid system; for example, coal particles are transported in pipelines using water as the suspending medium.

The field of *fluid dynamics* aims to describe qualitatively and quantitatively the flow that will develop in a given system. One might for example want to know how oil flows through a pipeline and what pressure difference is necessary to drive this flow; or how to design irrigation canals; or how a pollutant disperses in the atmosphere. Finding an answer to these questions often involves the interplay of theory and experiment. The theory consists of the basic physical laws governing the behavior of fluids in motion. These are well known laws of mechanics and comprise the expression of conservation of mass and of Newton's second law of motion. However, these laws give rise to complex partial differential equations that can only be solved exactly for very simple idealized situations. Hence, a formal theory often needs to be complemented by experimental observations and measurements.

The experimental approach is planned such that the results are as widely applicable as possible. This is achieved by using the concept of *similarity*, so that measurements made on one system (for example, in the laboratory) can be used to describe the behavior of other similar systems (outside the laboratory). *Dimensional analysis* is a systematic procedure that allows us to transfer knowledge about the characteristics of a flow from a small laboratory scale to a different scale on an industrial plant or in the natural environment.

The flow around a solid body immersed in the fluid is termed an *external flow*. The study of the external flow past an object allows us to determine the forces the fluid exerts on the object. Much of the information on external flows has been obtained from experiments carried out on scale models of the actual objects. Examples include the wind tunnel testing of small models of airplanes, buildings, and even entire cities. An important result of the analysis of a very simple external flow is the well-known equation for the velocity of settling of a small, solid sphere in a fluid, called *Stokes' settling velocity*.

In some circumstances it may be reasonable to make simplifying assumptions to the governing equations, so that analytical solutions may be obtained. For example, in some flows we may assume that the effect of viscosity is small and can be neglected. In other flows, particularly very slow ones, we may assume that acceleration of the fluid is negligible. These rather drastic assumptions greatly simplify the theoretical analysis and provide the opportunity to obtain detailed solutions to a variety of complex problems.

This contribution is an introduction to fluid dynamics. The emphasis is on understanding the fundamental concepts involved. Applications to various engineering and natural flows are discussed briefly for motivation.

2. The Continuum Hypothesis

A fluid is composed of a large number of molecules, widely spaced for a gas and more closely spaced for a liquid. In either case, the distance between the molecules is much larger than the molecular diameter. These molecules are in constant motion and undergoing collisions with one another. In classical fluid mechanics, we do not wish to describe such a microscopic scale. Instead, we aim to describe the macroscopic behavior of the fluid, that is, the average manifestation of the molecular motion. For example, when we say the velocity at a certain point in the fluid is 1 m/s, we are really referring to the average velocity of the molecules in a small volume surrounding the point.

But, is this an acceptable way to describe the behavior of a fluid? Let us answer this question by considering a simple experiment. Suppose we wish to measure the density of the air at a given point in a room. We start by taking a 'snapshot' showing the positions of all the molecules in the neighborhood of the point at a given instant in time. We then measure the mass of molecules of air δm within a given volume δV centered about the point, and calculate the density ρ of the air at the point by dividing the former by the latter. Figure 1 shows the results of such an experiment. If δV is relatively large, the calculated value of ρ is influenced by bulk variations, such as a temperature variation between the lower and upper levels in the room. If, on the other hand, δV is very small, it contains only a few molecules, and there are large swings in the calculated ρ as the boundary of δV passes the locations of the few molecules. We can define a limiting volume δV^* below which molecular variations are important and above which bulk variations may be important. The density of the air ρ at the point is defined by its limiting plateau value

$$\rho = \lim_{\delta V \to \delta V^*} \frac{\delta m}{\delta V} \tag{1}$$

The value δV^* is about 10^{-9} mm^3 for all liquids and for gases at atmospheric pressure. For air at standard conditions, such volume contains approximately 3×10^7 molecules, which is indeed a large number of molecules and the idea of using average values taken over this volume is reasonable.
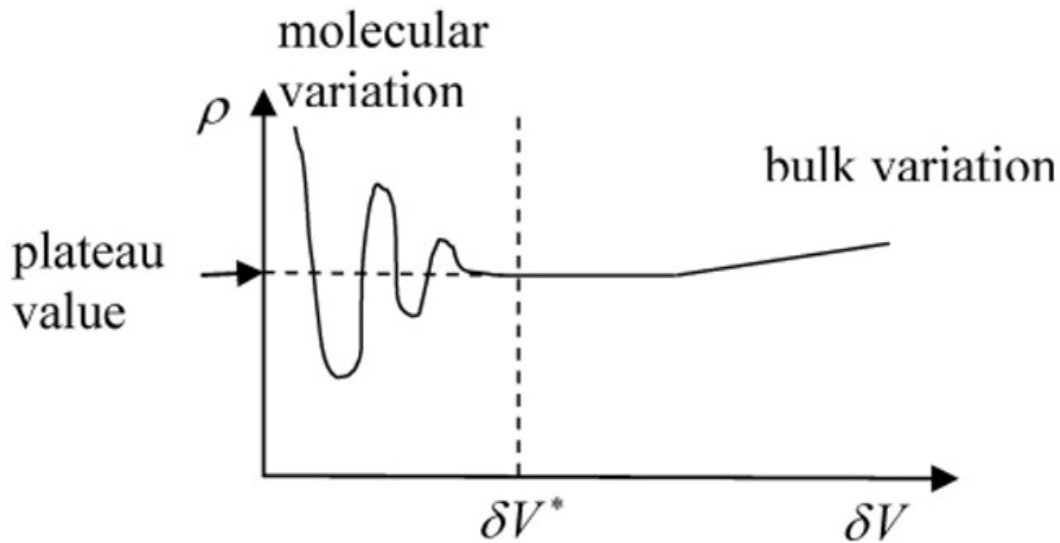

Figure 1. Calculated density versus the size of the sample volume, at a fixed location and at a given instant.

We shall therefore ignore the discrete molecular structure of the fluid and assume that all fluid characteristics we are interested in, such as velocity, pressure and density, vary continuously throughout the fluid. We treat the fluid as a *continuous medium* or *continuum*.

This continuum macroscopic approximation is valid for most engineering problems, as these are generally concerned with volume scales much larger than the limiting value. One area of fluid mechanics in which the continuum concept breaks down is in the study of gases at such low pressures that the mean free path is comparable to or larger than the physical size of the system, such as would be encountered in the upper altitudes of the atmosphere. A kinetic theory approach is necessary for studying the dynamics of these rarefied gases.

3. Conservation Equations

3.1. Conservation of Mass: The Continuity Equation

Consider a small parallelepiped in a Cartesian coordinate system, with volume $dV = dx\,dy\,dz$ and total surface area dS, as shown in Figure 2. This is an elemental

control volume, which is fixed in space. The surfaces of the control volume are open and fluid passes freely across them.
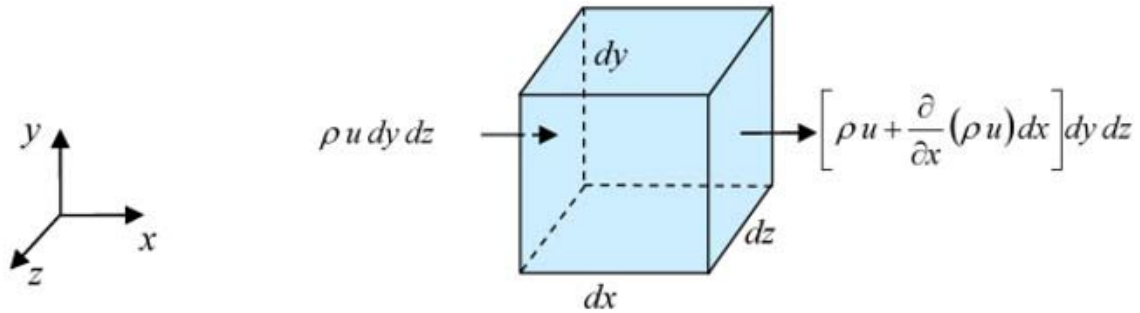

Figure 2. Elemental control volume. For clarity, only the mass flow in the *x* direction is shown.

The mass flow into and out of the volume can be calculated by considering the flow terms on all six faces, three inlets and three outlets. Thus, in the *x* direction, the rate of mass flow per unit area into the element on the left face is ρu and the rate of mass flow per unit area out of the element on the right face is a little different and given by

$$\rho u + \frac{\partial}{\partial x}(\rho u)dx.$$

Here *u* is the velocity of the fluid in the *x* direction. Similar terms may be derived for the *y* and *z* directions. The total rate of inflow and outflow of the control volume may be calculated by adding all the inlets and outlets, respectively. Hence,

$$\begin{array}{c} \text{rate of inflow of mass} \\ \text{through } dS \end{array} = \quad \rho u\,dy\,dz \quad + \quad \rho v\,dx\,dz \quad + \quad \rho w\,dx\,dy \quad (2)$$

and

$$\begin{array}{c} \text{rate of outflow of mass} \\ \text{through } dS \end{array} = \begin{array}{l} \left(\rho u + \dfrac{\partial(\rho u)}{\partial x}dx\right)dy\,dz + \\[2ex] \left(\rho v + \dfrac{\partial(\rho v)}{\partial y}dy\right)dx\,dz + \\[2ex] \left(\rho w + \dfrac{\partial(\rho w)}{\partial z}dz\right)dx\,dy \end{array} \quad (3)$$

where *v* and *w* are the velocities in the *y* and *z* directions, respectively. The amount of mass inside of the control volume can only change through a change in density, since the volume itself is fixed (*i.e.*, it cannot compress or expand). Thus,

$$\text{rate of accumulation of mass in } dV = \frac{\partial \rho}{\partial t} dV \qquad (4)$$

where t denotes time. Since in physical and chemical processes, mass is neither created nor destroyed, we expect that

$$\text{rate of accumulation of mass inside } dV = \text{rate of inflow of mass} - \text{rate of outflow of mass} \qquad (5)$$

This is the principle of conservation of mass.

Introducing Eqs. (2)-(4) into the conservation principle, Eq. (5), leads to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

We can re-write this equation using vector operators as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad (6)$$

where bold denotes a three-dimensional vector. This is a general statement of conservation of mass in a fluid system and is valid in any coordinate system. It states that the density in the neighborhood of a point in the fluid may change only through unbalanced flows in that region. It is valid for steady or unsteady, compressible or incompressible, and viscous or frictionless flows. It is commonly referred to as the *continuity equation* because it requires no assumptions other than that the velocity and density are continuum functions.

Two particular cases are of interest. For steady flow of a compressible fluid, we have

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

or

$$\nabla \cdot (\rho \mathbf{u}) = 0$$

since ρ is not a function of time, but can be a function of position.

For incompressible fluids, ρ is constant, and hence

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

or

$$\nabla \cdot \mathbf{u} = \text{div } \mathbf{u} = 0 \tag{7}$$

This is true regardless of whether the flow is steady or unsteady. This is effectively a statement of conservation of volume.

Many practical engineering flows are approximately incompressible, *i.e.*, $\partial\rho/\partial t \approx 0$, the main exceptions being the high-speed flows of gases and combustion systems where large temperature changes occur. Table 1 gives Eq. (7) in rectangular, cylindrical and spherical coordinates.

Rectangular coordinates	$\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0$
Cylindrical polar coordinates	$\dfrac{1}{r}\dfrac{\partial(r u_r)}{\partial r} + \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta} + \dfrac{\partial u_z}{\partial z} = 0$
Spherical coordinates	$\dfrac{1}{r^2}\dfrac{\partial(r^2 u_r)}{\partial r} + \dfrac{1}{r\sin\theta}\dfrac{\partial(u_\theta \sin\theta)}{\partial \theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial u_\phi}{\partial \phi} = 0$

Table 1. Continuity equation for a Newtonian, incompressible fluid.

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Biographical Sketch

Dr Silvana Cardoso has an undergraduate degree in Chemical Engineering from the University of Porto, Portugal and a PhD in Applied Mathematics and theoretical Physics from the University of Cambridge, UK.

In 1996, Dr Cardoso was appointed to the position of University Assistant Lecturer at the Department of Chemical Engineering, Cambridge. She was awarded an Esso Engineering Teaching Fellowship in 1999, promoted to University Lecturer in 2000 and promoted to University Senior Lecturer in 2003. In 2005, she was appointed to a Readership in Fluid Mechanics and the Environment at the University of Cambridge. Dr Cardoso's scientific research interests lie in the areas of fluid flow, mixing and chemical reaction in a wide variety of engineering and environmental processes. She has worked on a diverse range of projects, including

(i)   Bubbly flows and the triggering of volcanic eruptions.

(ii)   Mixing of fluids by small bubbles, with applications to waste-water treatment, destratification of natural lakes and reservoirs, and in CO_2 sequestration.

(iii)   Particle laden plumes and jets, with application in the sedimentation from industrial chimney.

(iv)   Volatile desorption during the solidification of melts with application in the casting of metals.

(v)   Phase change in subcooled liquids, with application to the design and operation of portable heat systems.

(vi)   Double diffusion and the dispersal of pollutants in ocean and estuary discharges.

(vii)   The development of natural convection in exothermal reactive systems.

Dr Cardoso's research work has been awarded numerous prizes, including the J. T. Knight prize, University of Cambridge (1994) and several IChemE Fluids Mixing Prizes (2001, 2002, 2005).